Mathematica 11.3 Integration Test Results

Test results for the 53 problems in "6.4.7 (d hyper)^m (a+b (c coth)^n)^p.m"

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1-\text{Coth}[x]^2}\, dx$$

Optimal (type 3, 3 leaves, 3 steps):

$$\text{ArcSin}[\text{Coth}[x]]$$

Result (type 3, 30 leaves):

$$\sqrt{-\text{Csch}[x]^2}\left(-\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]\right)\text{Sinh}[x]$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \left(1-\text{Coth}[x]^2\right)^{3/2} dx$$

Optimal (type 3, 24 leaves, 4 steps):

$$\frac{1}{2}\text{ArcSin}[\text{Coth}[x]]+\frac{1}{2}\text{Coth}[x]\sqrt{-\text{Csch}[x]^2}$$

Result (type 3, 51 leaves):

$$\frac{1}{8}\sqrt{-\text{Csch}[x]^2}\left(\text{Csch}\left[\frac{x}{2}\right]^2-4\,\text{Log}\left[\text{Cosh}\left[\frac{x}{2}\right]\right]+4\,\text{Log}\left[\text{Sinh}\left[\frac{x}{2}\right]\right]+\text{Sech}\left[\frac{x}{2}\right]^2\right)\text{Sinh}[x]$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[x]^2\sqrt{a+b\,\text{Coth}[x]^2}\, dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$-\frac{(a+2\,b)\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{2\sqrt{b}}+\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]-\frac{1}{2}\text{Coth}[x]\sqrt{a+b\,\text{Coth}[x]^2}$$

Result (type 3, 191 leaves):

$$-\left(\left(\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\text{Csch}[x]^2}\right.\right.$$
$$\left(\sqrt{2}\,\sqrt{a+b}\,(a+2b)\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]+\right.$$
$$\sqrt{b}\left(-2\sqrt{2}\,(a+b)\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]+\right.$$
$$\left.\left.\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\,\text{Coth}[x]\,\text{Csch}[x]\right)\right)$$
$$\left.\left.\text{Sinh}[x]\right)\Big/\left(2\sqrt{2}\,\sqrt{b}\,\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)\right)$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[x]\,\sqrt{a+b\,\text{Coth}[x]^2}\,dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]-\sqrt{a+b\,\text{Coth}[x]^2}$$

Result (type 3, 108 leaves):

$$\left(\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\,\text{Csch}[x]-\right.$$
$$\left.\frac{\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\text{Csch}[x]^2}{\sqrt{2}}\right)\Big/\left(\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\text{Csch}[x]^2}\right)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+b\,\text{Coth}[x]^2}\,dx$$

Optimal (type 3, 60 leaves, 6 steps):

$$-\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]+\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]$$

Result (type 3, 137 leaves):

$$\frac{1}{2}\left(-\sqrt{a+b}\,\text{Log}[1-\text{Coth}[x]]+\sqrt{a+b}\,\text{Log}[1+\text{Coth}[x]]-2\sqrt{b}\,\text{Log}\left[b\,\text{Coth}[x]+\sqrt{b}\,\sqrt{a+b\,\text{Coth}[x]^2}\right]-\sqrt{a+b}\,\text{Log}\left[a-b\,\text{Coth}[x]+\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^2}\right]+\sqrt{a+b}\,\text{Log}\left[a+b\,\text{Coth}[x]+\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^2}\right]\right)$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int\sqrt{a+b\,\text{Coth}[x]^2}\,\text{Tanh}[x]\,dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$-\sqrt{a}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a}}\right]+\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]$$

Result (type 3, 134 leaves):

$$\left(\left(\sqrt{-a}\,\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{-a}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}+\sqrt{b}\,\sqrt{a+b}\,\text{ArcSinh}\left[\frac{\sqrt{a+b}\,\text{Sinh}[x]}{\sqrt{b}}\right]\sqrt{\frac{-a+b+(a+b)\,\text{Cosh}[2x]}{b}}\right)\text{Csch}[x]\right)\Big/\left(\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\right)$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int\sqrt{a+b\,\text{Coth}[x]^2}\,\text{Tanh}[x]^2\,dx$$

Optimal (type 3, 48 leaves, 5 steps):

$$\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]-\sqrt{a+b\,\text{Coth}[x]^2}\,\text{Tanh}[x]$$

Result (type 3, 114 leaves):

$$\left(\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\left(2\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\text{Sinh}[x]-\sqrt{2}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\,\text{Tanh}[x]\right)\right)\Big/\left(2\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)$$

Problem 26: Result unnecessarily involves higher level functions and more than

twice size of optimal antiderivative.

$$\int \left(a+b\,\text{Coth}[x]^2\right)^{3/2}\text{Tanh}[x]\,dx$$

Optimal (type 3, 71 leaves, 8 steps):

$$-a^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a}}\right]+(a+b)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]-b\sqrt{a+b\,\text{Coth}[x]^2}$$

Result (type 4, 1088 leaves):

$$-b\sqrt{\frac{-a+b+a\,\text{Cosh}[2\,x]+b\,\text{Cosh}[2\,x]}{-1+\text{Cosh}[2\,x]}}+$$

$$\frac{1}{2}\left(-\left(\left(i\left(-3\,a^2+2\,a\,b+b^2\right)\left(1+\text{Cosh}[x]\right)\sqrt{\frac{-1+\text{Cosh}[2\,x]}{\left(1+\text{Cosh}[x]\right)^2}}\sqrt{\frac{-a+b+\left(a+b\right)\text{Cosh}[2\,x]}{-1+\text{Cosh}[2\,x]}}\right.\right.\right.$$

$$\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2\,a+b+2\sqrt{a\left(a+b\right)}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right],\frac{2\,a+b+2\sqrt{a\left(a+b\right)}}{2\,a+b-2\sqrt{a\left(a+b\right)}}\right]-\right.$$

$$2\,\text{EllipticPi}\left[\frac{2\,a+b+2\sqrt{a\left(a+b\right)}}{b},\,i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2\,a+b+2\sqrt{a\left(a+b\right)}}}\,\text{Tanh}\left[\frac{x}{2}\right]\right],\right.$$

$$\left.\left.\frac{2\,a+b+2\sqrt{a\left(a+b\right)}}{2\,a+b-2\sqrt{a\left(a+b\right)}}\right]\right)\text{Tanh}\left[\frac{x}{2}\right]\sqrt{\frac{2\,a+b+2\sqrt{a\left(a+b\right)}+b\,\text{Tanh}\left[\frac{x}{2}\right]^2}{2\,a+b+2\sqrt{a\left(a+b\right)}}}$$

$$\left.\sqrt{1+\frac{b\,\text{Tanh}\left[\frac{x}{2}\right]^2}{2\,a+b-2\sqrt{a\left(a+b\right)}}}\right)\Bigg/\left(\sqrt{\frac{b}{2\,a+b+2\sqrt{a\left(a+b\right)}}}\sqrt{-a+b+\left(a+b\right)\text{Cosh}[2\,x]}\right.$$

$$\left.\left.\left.\sqrt{\text{Tanh}\left[\frac{x}{2}\right]^2}\left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)\sqrt{\frac{4\,a\,\text{Tanh}\left[\frac{x}{2}\right]^2+b\left(1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}}\right)\right)+\right.$$

$$\frac{1}{\sqrt{-a+b+\left(a+b\right)\text{Cosh}[2\,x]}}\,3\left(a^2+2\,a\,b+b^2\right)\sqrt{-1+\text{Cosh}[2\,x]}\sqrt{\frac{-a+b+\left(a+b\right)\text{Cosh}[2\,x]}{-1+\text{Cosh}[2\,x]}}$$

$$\left(-\left(\left(i\left(1+\text{Cosh}[x]\right)\sqrt{\frac{-1+\text{Cosh}[2\,x]}{\left(1+\text{Cosh}[x]\right)^2}}\left(\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{2\,a+b+2\sqrt{a\left(a+b\right)}}}\right.\right.\right.\right.\right.\right.$$

$$\text{Tanh}\left[\frac{x}{2}\right]\Big],\ \frac{2a+b+2\sqrt{a(a+b)}}{2a+b-2\sqrt{a(a+b)}}\Big] - 2\,\text{EllipticPi}\Big[\frac{2a+b+2\sqrt{a(a+b)}}{b},$$

$$i\,\text{ArcSinh}\Big[\sqrt{\frac{b}{2a+b+2\sqrt{a(a+b)}}}\ \text{Tanh}\left[\frac{x}{2}\right]\Big],\ \frac{2a+b+2\sqrt{a(a+b)}}{2a+b-2\sqrt{a(a+b)}}\Big]\Big)$$

$$\text{Tanh}\left[\frac{x}{2}\right]\sqrt{\frac{2a+b+2\sqrt{a(a+b)}+b\,\text{Tanh}\left[\frac{x}{2}\right]^2}{2a+b+2\sqrt{a(a+b)}}}\ \sqrt{1+\frac{b\,\text{Tanh}\left[\frac{x}{2}\right]^2}{2a+b-2\sqrt{a(a+b)}}}\Big)\Big/$$

$$\Big(\sqrt{\frac{b}{2a+b+2\sqrt{a(a+b)}}}\ \sqrt{-1+\text{Cosh}[2x]}\ \sqrt{\text{Tanh}\left[\frac{x}{2}\right]^2}$$

$$\left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)\sqrt{\frac{4a\,\text{Tanh}\left[\frac{x}{2}\right]^2+b\left(1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}{\left(-1+\text{Tanh}\left[\frac{x}{2}\right]^2\right)^2}}\Big)\Big) +$$

$$\Big(4\sqrt{2b+a\,(-1+\text{Cosh}[2x])+b\,(-1+\text{Cosh}[2x])}$$

$$\Big(-\frac{\text{ArcTanh}\Big[\frac{\sqrt{a}\,\sqrt{-1+\text{Cosh}[2x]}}{\sqrt{a\,(-1+\text{Cosh}[2x])+b\,(1+\text{Cosh}[2x])}}\Big]}{\sqrt{a}}+\frac{1}{\sqrt{a+b}}\text{Log}\Big[a\sqrt{-1+\text{Cosh}[2x]}+b$$

$$\sqrt{-1+\text{Cosh}[2x]}+\sqrt{a+b}\ \sqrt{a\,(-1+\text{Cosh}[2x])+b\,(1+\text{Cosh}[2x])}\Big]\Big)\ \text{Sinh}[x]^2$$

$$\text{Sinh}[2x]\ \text{Tanh}[x]\Big)\Big/\Big(3\,(-1+\text{Cosh}[2x])^2\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\Big)\Big)\Big)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int\left(a+b\,\text{Coth}[x]^2\right)^{3/2}\text{Tanh}[x]^2\,dx$$

Optimal (type 3, 77 leaves, 7 steps):

$$-b^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]+(a+b)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]-a\sqrt{a+b\,\text{Coth}[x]^2}\,\text{Tanh}[x]$$

Result (type 3, 180 leaves):

$$\left(\left(-\sqrt{2}\,b^{3/2}\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\text{Cosh}[x]+\sqrt{2}\,(a+b)^2\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\text{Cosh}[x]-a\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\,\text{Tanh}[x]\right)\Big/\left(\sqrt{2}\,\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int\left(1+\text{Coth}[x]^2\right)^{3/2}dx$$

Optimal (type 3, 50 leaves, 6 steps):

$$-\frac{5}{2}\,\text{ArcSinh}[\text{Coth}[x]]+2\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Coth}[x]}{\sqrt{1+\text{Coth}[x]^2}}\right]-\frac{1}{2}\,\text{Coth}[x]\sqrt{1+\text{Coth}[x]^2}$$

Result (type 3, 116 leaves):

$$-\frac{1}{8}\left(1+\text{Coth}[x]^2\right)^{3/2}\text{Sech}[2x]^2\left(16\,\text{ArcTanh}\left[\frac{\text{Cosh}[x]}{\sqrt{\text{Cosh}[2x]}}\right]\sqrt{\text{Cosh}[2x]}\,\text{Sinh}[x]^3+4\left(\text{ArcTan}\left[\frac{\text{Cosh}[x]}{\sqrt{-\text{Cosh}[2x]}}\right]\sqrt{-\text{Cosh}[2x]}-4\sqrt{2}\,\sqrt{\text{Cosh}[2x]}\,\text{Log}\left[\sqrt{2}\,\text{Cosh}[x]+\sqrt{\text{Cosh}[2x]}\right]\right)\text{Sinh}[x]^3+\text{Sinh}[4x]\right)$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Coth}[x]^3}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 47 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}-\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{b}$$

Result (type 3, 98 leaves):

$$\frac{1}{2}\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\text{Csch}[x]^2}\left(-\frac{\sqrt{2}}{b}+\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\text{Sinh}[x]}{\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right)$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Coth}[x]^2}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 60 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{\sqrt{b}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{\sqrt{a+b}}$$

Result (type 3, 134 leaves):

$$\left(\left(-\sqrt{a+b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]+\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\right)\right.$$
$$\left.\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\text{Csch}[x]^2}\,\text{Sinh}[x]\right)\Big/\left(\sqrt{b}\,\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 29 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}$$

Result (type 3, 82 leaves):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\,\text{Csch}[x]}{\sqrt{a+b}\,\sqrt{\left(-a+b+(a+b)\,\text{Cosh}[2x]\right)\text{Csch}[x]^2}}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\ \text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{\sqrt{a+b}}$$

Result (type 3, 83 leaves):

$$\frac{1}{2\sqrt{a+b}}\left(-\text{Log}[1-\text{Coth}[x]]+\text{Log}[1+\text{Coth}[x]]-\text{Log}\left[a-b\,\text{Coth}[x]+\sqrt{a+b}\ \sqrt{a+b\,\text{Coth}[x]^2}\right]+\text{Log}\left[a+b\,\text{Coth}[x]+\sqrt{a+b}\ \sqrt{a+b\,\text{Coth}[x]^2}\right]\right)$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 56 leaves, 7 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a}}\right]}{\sqrt{a}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Coth}[x]^2}}{\sqrt{a+b}}\right]}{\sqrt{a+b}}$$

Result (type 3, 127 leaves):

$$-\left(\left(\left(\frac{\text{ArcTan}\left[\frac{\sqrt{2}\ \sqrt{-a}\ \text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]}{\sqrt{-a}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\ \sqrt{a+b}\ \text{Sinh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]}{\sqrt{a+b}}\right)\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\ \text{Csch}[x]\right)\Big/\left(\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\right)\right)$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[x]^2}{\sqrt{a+b\,\text{Coth}[x]^2}}\,dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\ \text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{\sqrt{a+b}}-\frac{\sqrt{a+b\,\text{Coth}[x]^2}\ \text{Tanh}[x]}{a}$$

Result (type 3, 126 leaves):

$$\left(\left(\sqrt{2}\, a\, \text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right]\text{Cosh}[x]-\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)\right.$$
$$\left.\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\,\text{Tanh}[x]\right)\Big/\left(\sqrt{2}\, a\,\sqrt{a+b}\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}\right)$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]^2}{\left(a+b\,\text{Coth}[x]^2\right)^{3/2}}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b}\,\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^2}}\right]}{(a+b)^{3/2}}-\frac{\text{Coth}[x]}{(a+b)\,\sqrt{a+b\,\text{Coth}[x]^2}}$$

Result (type 3, 135 leaves):

$$\left(\left(-2\sqrt{a+b}\,\text{Cosh}[x]\,\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}+\right.\right.$$
$$\left.\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\sqrt{a+b}\,\text{Cosh}[x]}{\sqrt{-a+b+(a+b)\,\text{Cosh}[2x]}}\right](-a+b+(a+b)\,\text{Cosh}[2x])\right)$$
$$\left.\sqrt{(-a+b+(a+b)\,\text{Cosh}[2x])\,\text{Csch}[x]^2}\,\text{Sinh}[x]\right)\Big/$$
$$\left(\sqrt{2}\,(a+b)^{3/2}\,(-a+b+(a+b)\,\text{Cosh}[2x])^{3/2}\right)$$

Problem 51: Unable to integrate problem.

$$\int \text{Coth}[x]\,\sqrt{a+b\,\text{Coth}[x]^4}\,dx$$

Optimal (type 3, 89 leaves, 8 steps):

$$-\frac{1}{2}\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Coth}[x]^2}{\sqrt{a+b\,\text{Coth}[x]^4}}\right]+$$
$$\frac{1}{2}\sqrt{a+b}\,\text{ArcTanh}\left[\frac{a+b\,\text{Coth}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^4}}\right]-\frac{1}{2}\sqrt{a+b\,\text{Coth}[x]^4}$$

Result (type 8, 17 leaves):

$$\int \text{Coth}[x]\,\sqrt{a+b\,\text{Coth}[x]^4}\,dx$$

Problem 52: Unable to integrate problem.

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^4}}\,dx$$

Optimal (type 3, 40 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{a+b\,\text{Coth}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^4}}\right]}{2\,\sqrt{a+b}}$$

Result (type 8, 17 leaves):

$$\int \frac{\text{Coth}[x]}{\sqrt{a+b\,\text{Coth}[x]^4}}\,dx$$

Problem 53: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[x]}{\left(a+b\,\text{Coth}[x]^4\right)^{3/2}}\,dx$$

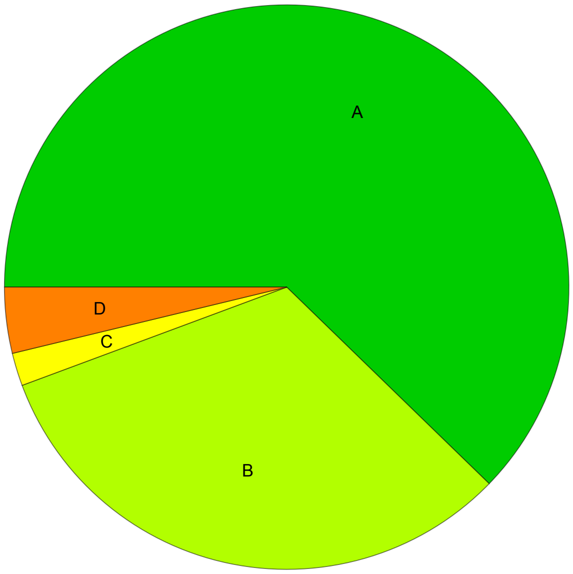
Optimal (type 3, 74 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{a+b\,\text{Coth}[x]^2}{\sqrt{a+b}\,\sqrt{a+b\,\text{Coth}[x]^4}}\right]}{2\,(a+b)^{3/2}}-\frac{a-b\,\text{Coth}[x]^2}{2\,a\,(a+b)\,\sqrt{a+b\,\text{Coth}[x]^4}}$$

Result (type 3, 31 578 leaves): Display of huge result suppressed!

Summary of Integration Test Results

53 integration problems



A - 33 optimal antiderivatives

B - 17 more than twice size of optimal antiderivatives

C - 1 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts